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Hydrodynamics and heat transfer of turbulent gas suspension flows in tubes—1. Hydrodynamics

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Abstract—By the method of averaging over the ensemble of turbulent flow realizations, the averaged motion and mass balance equations for a solid phase and a flow as a whole are derived. Closed expressions for the second single-point moments of fluctuations of the solid and carrier characteristics are obtained in terms of correlations of the carrier phase velocity fluctuations in a non-homogeneous turbulent flow. Based on these expressions, a set of equations is written for the second single-point moments of the carrier phase velocity fluctuations in the presence of particles. Hydrodynamic analysis is presented for turbulent gas suspension flows in tubes. Comparison with experimental data shows a satisfactory description of the processes of momentum transfer by a particle-laden flow.

1. INTRODUCTION

A TWO-PHASE flow involving a solid dispersed phase is predominantly turbulent. The explanation lies in the fact that a laminar flow has a very limited suspension-carrying capacity and, being laden with particles, it exhibits a very early transition to turbulent flow [1, 2]. The theory of the turbulent disperse flow has been worked out to a much lesser extent than the theory of a turbulent single-phase flow. Full analysis of publications concerned with the methods for describing and predicting turbulent disperse flows is not within the scope of this paper; there are a variety of monographs which deal with two-phase turbulent flows and survey relevant studies [3–7].

The particles of the admixture in a turbulent flow can be conventionally divided into large and small depending on their behaviour in a turbulent flow. Large particles, the dynamic relaxation time of which considerably exceeds the time scale of turbulence, are practically not entrained into the carrier phase pulsating motion. Characteristic for these particles are the effects of the velocity slip of phases, collision of particles with the channel walls and with each other; the motion of large particles is influenced by gravitation and by the Magnus force arising due to the rotation of particles in shear flow. Large particles can influence the pulsating structure of flow in the cases of a perceptible velocity slip of phases, formation turbulent wake behind the particles, destruction of a viscous sublayer, etc. In the case of low averaged velocity slips of phases (e.g. in long tubes in the absence of gravity), the effect of large particles on the flow characteristics is insignificant, since they are not involved in pulsating motion, cannot therefore exert an inverse effect upon the carrier phase and, consequently, upon the characteristics of the flow as a whole. The coarseness of the admixture particles is determined by their relative size, by the material density ratio of the particles and carrier phase and also by the Reynolds number of the main stream [3].

The behaviour of small particles, the dynamic relaxation time of which is comparable with, or smaller than, the life of energy-carrying moles (turbulent time scale), depends on the intensity of the pulsating motion of the carrier phase and, in turn, determines the influence of the admixture on the main stream characteristics. In the case of small particles, the averaged velocity slip of phases is virtually absent and, therefore, the hydrodynamics and heat transfer of a disperse flow can be described within the framework of single-velocity approximation. The factors governing the behaviour of large particles are insignificant for the analysis of the behaviour of small particles. Important for the latter are the effects associated with the interphase interaction in pulsating slip of phase and with the nonuniformity of the field of the turbulent carrier phase fluctuations (turbulent migration [8, 9]), and also the turbulent diffusion of an admixture. In what follows the consideration will be restricted to the hydrodynamics of disperse flows with an admixture of small particles with low volume concentration.

The effect of particles on the intensity of momentum and heat transfer in a turbulent disperse flow is determined by two factors: the degree with which the particles are entrained into pulsating motion of the medium and the character of the inverse effect of the admixture on the carrier phase velocity fluctuations. The problem of determining the degree of the entrainment of particles into the turbulent flow pulsating motion, i.e. determining the correlation between velocity fluctuations of the particles and carrier phase was the concern of a number of publications [3–7, 10, 11]. However, the relations obtained in these works for the particle velocity fluctuation correlation

NOMENCLATURE

- a — particle radius
- $C, \langle C \rangle$ — actual and averaged volume concentrations of solid phase
- D_{ik} — tensor of turbulent diffusion of admixture particles
- E — turbulent energy of carrier phase, $(\langle u_x^2 \rangle + \langle u_y^2 \rangle + \langle u_z^2 \rangle)/2$
- $\bar{E}$ — pulsating energy of carrier phase normalized to average velocity of flow
- F_{ij}^u, F_u — two-time correlation functions of velocity fluctuations of non-stationary and stationary carrier phase flows
- $f_{u1}, f_{u2}, f_{u3}, f_{u4}$ — functions describing the effect of particles on the intensity of carrier phase turbulent fluctuations
- L — spatial scale of turbulence
- N — number of particles per flow volume
- $P, \langle P \rangle, p$ — actual, averaged and pulsating pressure in liquid phase, respectively
- R — channel radius
- Re — main stream Reynolds number, $2RU_m/\nu_1$
- T_E — time macroscale of turbulence
- T_p — time of interaction between particle and turbulent mole
- T_0 — characteristic time of change of averaged quantities
- $U_i, \langle U_i \rangle, u_i$ — actual, averaged and pulsating velocities of carrier phase, respectively
- U_m — average velocity of carrier phase
- $\bar{U}$ — averaged velocity of carrier phase normalized to average velocity of flow
- $V_i, \langle V_i \rangle, v_i$ — actual, averaged and pulsating velocities of solid phase, respectively
- V_{pk} — velocity of pth particles
- x, x_k — Cartesian coordinates
- x, r — longitudinal and radial coordinates of cylindrical coordinate system
- y — distance from tube wall normalized to tube radius, $1-r$.

Greek symbols

- δ — Dirac delta function
- δ_{ik} — Kronecker delta
- ε_{ij}^u — term describing interphase interaction in equation for second single-point moments of carrier phase velocity fluctuations
- μ_1 — dynamic viscosity of the liquid phase, $\rho_1 \nu_1$
- ν_1 — kinematic viscosity of the carrier phase
- ν_{t1}, ν_{t2} — turbulent kinematic viscosities of carrier and solid phases, respectively
- ν_{t1}^0 — turbulent kinematic viscosity of particle-free carrier phase
- ξ, ξ_0 — friction factor of gas suspension flow and single-phase gas flow, respectively
- ρ_1, ρ_2 — density of carrier phase and particle material, respectively
- τ_u — time of dynamic relaxation of particles, $(2/9)\,(\rho_2/\rho_1)\,(a^2/\nu_1)$
- $\langle \phi \rangle$ — mass concentration of admixture, $\rho_2/\rho_1 \langle C \rangle$
- ω — volume of particles, $4/3\pi a^3$
- Ω_N — volume of flow containing N particles
- Ω_u — parameter of dynamic relaxation of particles, τ_u/T_E.

moments and expressed in terms of the medium velocity fluctuation moments are valid only for steady-state homogeneous turbulence.

The problem of the inverse influence of particles on the carrier phase velocity fluctuations was also discussed in many publications. Most of the investigations revealed a reduction in the flow turbulence on introduction of fine particles. Thus, based on the energy or momentum balance between a turbulent mole and particles, the authors of refs. [12, 13] derived simple equations showing a decrease in the intensity of medium velocity fluctuations with an increase in the admixture weight concentration. In ref. [14], the reduction of the turbulent flow hydraulic resistance in the presence of particles is explained with the aid of a model based on complete suppression of vortices the scale of which is smaller than the size of particles. Additional dissipation of turbulence energy as a result of incomplete entrainment of particles in pulsating motion of the medium is determined in refs. [15, 16]. The degree of reduction in the carrier phase fluctuation intensity owing to additional turbulent energy dissipation induced by pulsating slip is estimated in refs. [7, 17] on the basis of the turbulence energy balance equation. In contrast to refs. [7, 12–17], it is assumed in ref. [18] on the basis of the experimental data of ref. [19], that along with dissipation of turbulent energy, the particles contribute to the generation of turbulence due to their entrainment into pulsating motion. The latter is also confirmed by investigations of the stability of laminar flows carrying solid disperse admixtures [2, 20] which reveal that particles not only lead to the degradation of high-frequency disturbances, but also cause the growth of low-frequency (most energy-intensive) impulses because of an increase in the origination of disturbances from averaged motion. Thus, the question of the effect of fine particles on the pulsating characteristics of a turbulent shear flow has not been resolved.

To calculate the hydrodynamics of disperse flows, various models of turbulent transfer are used within the framework of which the influence of particles on turbulence is taken into account, as, for instance, the Prandtl model in ref. [12] and different modifications of the van Driest models in refs. [7, 18]. However, the most considerable promise is offered by models which are based on equations for second single-point moments of velocity fluctuations because they provide the possibility to most consecutively take into account the effect of particles on turbulence. This approach was used for the first time in ref. [21] to analyse the effect of heavy inertia-free particles on a horizontal turbulent flow. Subsequently the second moment equations were used to calculate a turbulent disperse flow in channels [17] and in jets [22, 23]. However, the main attention in these studies was paid to the dissipative influence of a discrete admixture on turbulence. The authors of ref. [17] discarded the terms involving the solid phase velocity fluctuations, thus considerably overstating the additional liquid-phase turbulent energy dissipation due to pulsating slip, while the authors of refs. [22, 23] achieved closure through the set of equations for the second single-point moments of solid and carrier phase velocity fluctuations which were derived for single-phase steady-state turbulence. This solution of the closure problem does not take a full account of the influence of admixture on the pulsating structure of flow.

In the present study, using the method of averaging over the ensemble of turbulent flow realizations, expressions for the second single-point moments of fluctuations of the solid and carrier phase characteristics in a non-uniform turbulent flow were found from the equation of motion of a single-solid particle. Based on these expressions, a set of equations was derived for the second single-point moments of the carrier phase velocity fluctuations in the presence of a particle to calculate the hydrodynamics of gas suspension flows in tubes. The calculations neglected a change in the space scale of the carrier phase turbulence in the presence of particles. The modelling of the discrete admixture effect on the spatial pulsatory structure of a carrying flow is the purpose of further investigations.

2. AVERAGED MASS AND MOMENTUM BALANCE EQUATIONS FOR THE SOLID PHASE AND GAS SUSPENSION FLOW AS A WHOLE

The following restrictions are imposed on a two-phase system 'gas–particles': (1) the volume concentration of the admixture is so insignificant that the collision of particles with each other may be neglected; (2) the discrete phase is represented by solid non-deformable equally sized particles; (3) the Reynolds number based on the particle radius and flow fluctuational velocity is smaller than unity; (4) the gravity force is ignored which is justifiable at flow velocities much in excess of the free-fall velocity of particles; (5) the particles are assumed to be so small that the effects stemming from the rotation of particles could be neglected; (6) the collisions of particles with the channel wall and with each other are not taken into account.

Since the solid particles suspended in a turbulent gas flow are small ($\rho_2 \gg \rho_1$, $R/a \sim 10^3$), it is possible to regard them as pin-point particles and to describe the distribution of particles in the flow volume by the function

$$C(x,t) = \frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t)). \quad (1)$$

The solid phase velocity in Euler's notation is

$$V_i(x,t)C(x,t) = \frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))V_{pi}(t) \quad (2)$$

$$\frac{\mathrm{d}R_{pi}(t)}{\mathrm{d}t} = V_{pi}(t). \quad (3)$$

Differentiating equation (1) with respect to time and using equations (2) and (3) gives the solid phase mass balance equation

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_k}V_k C = 0. \quad (4)$$

In Stokes' approximation, the equation of single particle motion is

$$\frac{\mathrm{d}V_{pi}}{\mathrm{d}t} = \frac{1}{\tau_u}(U_i(R_p(t),t) - V_{pi}). \quad (5)$$

By imposing the approximation of point particles, the equation of motion for the carrier phase laden with particles can be written in the form

$$\frac{\partial U_i}{\partial t} + U_k\frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho_1}\frac{\partial P}{\partial x_i} + \nu_1\frac{\partial^2 U_1}{\partial x_k \partial x_k} - \frac{\rho_2}{\rho_1}\frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))\frac{\mathrm{d}V_{pi}}{\mathrm{d}t}. \quad (6)$$

From the quantities that characterize the motion of the carrier phase, separate the averaged and fluctuational one (the averaging is carried out over the turbulent flow realizations)

$$U_i(x,t) = \langle U_i(x,t)\rangle + u_i(x,t), \langle u_i(x,t)\rangle = 0$$

$$P(x,t) = \langle P(x,t)\rangle + p(x,t), \langle p(x,t)\rangle = 0.$$

The averaging of equations (1) and (2) over the turbulent flow realizations yields the averaged volume concentration and velocity of the solid phase

$$\langle C(x,t)\rangle = \left\langle \frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))\right\rangle$$

$$\langle C(x,t)\rangle\langle V_i(x,t)\rangle = \left\langle \frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))V_{pi}(t)\right\rangle.$$

The fluctuational solid phase velocity component is determined from the relation

$$V_i(x,t) = \langle V_i(x,t)\rangle + v_i(x,t)$$

so that the following equality is satisfied:

$$\langle C(x,t)v_i(x,t)\rangle = 0. \quad (7)$$

The averaging of equation (4) with regard for equation (7) results in the solid phase mass balance equation

$$\frac{\partial\langle C\rangle}{\partial t} + \frac{\partial}{\partial x_k}\langle C\rangle\langle V_k\rangle = 0. \quad (8)$$

Similarly, equations (5), (7) and (8) yield the solid phase motion equation

$$\langle C\rangle\left(\frac{\partial\langle V_i\rangle}{\partial t} + \langle V_k\rangle\frac{\partial\langle V_i\rangle}{\partial x_k}\right) = -\langle C\rangle\frac{\partial\langle v_i v_k\rangle}{\partial x_k}$$

$$-\langle v_i v_k\rangle\frac{\partial\langle C\rangle}{\partial x_k} + \frac{1}{\tau_u}\langle C u_i\rangle + \frac{\langle C\rangle}{\tau_u}(\langle U_i\rangle - \langle V_i\rangle). \quad (9)$$

The first term on the right-hand side of equation (9) accounts for the contribution of particles into the solid phase momentum transfer due to their entrainment into pulsating motion, which corresponds to the appearance of turbulent viscosity in this phase; the second term determines the diffusion force arising due to the concentration gradient; the last two terms relate directly to the interphase interaction.

By averaging equation (6) and adding it to equation (9), the momentum balance equation for the flow as a whole is obtained as

$$\frac{\partial\langle U_i\rangle}{\partial t} + \langle U_k\rangle\frac{\partial\langle U_i\rangle}{\partial x_k} + \frac{\rho_2}{\rho_1}\langle C\rangle\left(\frac{\partial\langle V_i\rangle}{\partial t}\right.$$

$$\left.+\langle V_k\rangle\frac{\partial\langle V_i\rangle}{\partial x_k}\right) = -\frac{1}{\rho_1}\frac{\partial\langle P\rangle}{\partial x_i} + \frac{\partial}{\partial x_k}\left[\nu_1\frac{\partial\langle U\rangle}{\partial x_k}\right.$$

$$\left.+\langle u_i u_k\rangle + \frac{\rho_2}{\rho_1}\langle C\rangle\langle v_i v_k\rangle\right]. \quad (10)$$

It is seen from equation (10) that the particles contribute not only to averaged convective transfer of momentum, but also to the turbulent stresses of the flow as a result of their entrainment into pulsating motion.

3. EQUATIONS FOR THE SECOND MOMENTS OF VELOCITY FLUCTUATIONS OF THE CARRIER PHASE LADEN WITH PARTICLES

Equations for the second single-point moments of the carrier phase velocity fluctuations in the presence of particles result from equation (6) and coincide with respective equations for a single-phase flow, except for the additional term which is associated with the interphase interaction and which has the form

$$\varepsilon^u_{ij} = \frac{\rho_2}{\rho_1}\left\langle\frac{\omega}{\Omega_N}\sum_{p=1}^{N}\delta(x-R_p(t))\right.$$

$$\left.\times\left[\frac{dV_{pi}}{dt}u_j(x,t) + \frac{dV_{pi}}{dt}u_i(x,t)\right]\right\rangle. \quad (11)$$

Substituting the single particle motion equation (5) and equation (11), obtain after averaging

$$\varepsilon^u_{ij} = \frac{\rho_2}{\rho_1}\frac{\langle C\rangle}{\tau_u}[2\langle u_i u_j\rangle - \langle u_i v_j\rangle - \langle u_j v_i\rangle]$$

$$+\frac{\rho_2}{\rho_1}\frac{1}{\tau_u}[\langle C u_i\rangle(\langle U_j\rangle - \langle V_j\rangle)$$

$$+\langle C u_j\rangle(\langle U_i\rangle - \langle V_i\rangle)]. \quad (12)$$

Equation (12) contains the single-point correlations of solid and liquid phase velocity fluctuations and also the correlations of the carrier phase velocity fluctuations and of particle concentration. Moreover, the second moments of solid phase velocity fluctuations are incorporated in the motion equations for the solid phase and for the flow as a whole. Therefore, the objective arises to express the second single-point moments of the fluctuations of solid and carrier phase characteristics in a non-uniform turbulent flow in terms of the correlations of velocity fluctuations of the carrier phase alone.

Assume that the intensity of the solid phase pulsating motion is primarily determined by the force of resistance arising during the fluctuation slip of phases; then the equation for solid phase velocity fluctuations will take the form

$$\frac{\partial v_i}{\partial t} + \langle V_k\rangle\frac{\partial v_i}{\partial x_k} + v_k\frac{\partial\langle V_i\rangle}{\partial x_k}$$

$$+\frac{\partial(v_i v_k - \langle v_i v_k\rangle)}{\partial x_k} = \frac{1}{\tau_u}(u_i - v_i). \quad (13)$$

At a zero initial velocity of the particle, equation (13) can be presented in the integral form as

$$v_i(x,t) = \frac{1}{\tau_u}\int_0^t \exp\left(-\frac{t-s}{\tau_u}\right)\left\{u_i(x,s)\right.$$

$$-\tau_u\left[\langle V_k(x,s)\rangle\frac{\partial v_i(x,s)}{\partial x_k}\right.$$

$$+v_k(x,s)\frac{\partial\langle V_i(x,s)\rangle}{\partial x_k}$$

$$+\frac{\partial}{\partial x_k}v_i(x,s)v_k(x,s)$$

$$\left.\left.-\langle v_i(x,s)v_k(x,s)\rangle\right]\right\}ds. \quad (14)$$

Calculate the second single-point moment of the solid and carrier phase velocity fluctuations. Multiplying equation (14) by $u_j(x,t)$, averaging and employing the substitution of variables in the integral, obtain

$$\langle u_j(x,t)v_i(x,t)\rangle$$
$$= \frac{1}{\tau_u}\int_0^t \exp\left(-\frac{S}{\tau_u}\right)\left\{\langle u_i(x,t-s)u_j(x,t)\rangle - \tau_u\left[\langle V_k(x,t-s)\rangle\left\langle u_j(x,t)\frac{\partial v_i(x,t-s)}{\partial x_k}\right\rangle + \langle u_j(x,t)v_k(x,t-s)\rangle\frac{\partial\langle V_i(x,t-s)\rangle}{\partial x_k} + \left\langle u_j(x,t)\frac{\partial}{\partial x_k}v_i(x,t-s)v_k(x,t-s)\right\rangle\right]\right\}\mathrm{d}s. \quad (15)$$

Determine the correlation moment of the carrier phase velocity fluctuations

$$\langle u_i(x,t-s)u_j(x,t)\rangle = F^u_{ij}(t-s/2,s) = F^u_{ij}(t,s) - \frac{s}{2}\frac{\partial F^u_{ij}(t,s)}{\partial t} \approx F_u(s)\langle u_i(x,t)u_j(x,t)\rangle - \frac{s}{2}F_u(s)\frac{\partial}{\partial t}\langle u_i(x,t)u_j(x,t)\rangle \quad (16)$$

where $F_u(s)$ is the two-time correlation function of the carrier phase velocity fluctuations. The value of F^u_{ij} changes significantly when the first argument is changed by the value of the order of T_0, and the second argument is changed by the value of the order of T_E, where T_0 is the characteristic time for the variation of averaged parameters, T_E is the time macroscale of turbulence (the life of an energy-carrying mole). It is assumed in what follows that $T_E \ll T_0$ and, consequently, the first and second arguments in F^u_{ij} are the slow and fast variables, respectively. It is this fact which allowed the restriction to the first term in the expansion of F^u_{ij} in the second argument (16).

It follows from equation (15) that the ratio of the first term within braces to the terms describing convective and diffusive transfer [to the first and second bracketed terms in equation (15)] is proportional to τ_u/T_0. Then, imposing the condition $\tau_u/T_0 \ll 1$ [and, consequently, considering the cases $\tau_u/T_E = O(1)$], it is possible to confine the discussion to the first terms in equations (16) and (15) when calculating the convective and diffusive terms between the brackets in equation (15). Relevant calculations in equation (15) yield a closed relation for the second single-point moment of solid and carrier phase velocity fluctuations accurate to the terms of the order of $(\tau_u/T_0)^2$ expressed in terms of velocity fluctuation moments of the carrier phase alone

$$\langle u_i(x,t)v_j(x,t)\rangle = f_{u1}\langle u_iu_j\rangle - \tau_u f_{u2}\left(\frac{1}{2}\frac{\partial\langle u_ju_k\rangle}{\partial t} + \langle V_k\rangle\left\langle u_i\frac{\partial u_j}{\partial x_k}\right\rangle + \langle u_iu_k\rangle\frac{\partial\langle V_j\rangle}{\partial x_k} + \left\langle u_i\frac{\partial u_ju_k}{\partial x_k}\right\rangle\right) \quad (17)$$

where the coefficients f_{u1} and f_{u2} characterize the degree with which the particles are entrained into pulsating motion. They are determined by the formulae

$$f_{u1} = \frac{1}{\tau_u}\int_0^\infty \exp\left(-\frac{s}{\tau_u}\right)F_u(s)\,\mathrm{d}s$$
$$f_{u2} = \frac{1}{\tau_u^2}\int_0^\infty s\exp\left(-\frac{s}{\tau_u}\right)F_u(s)\,\mathrm{d}s. \quad (18)$$

Similarly, the second single-point moment of the discrete phase velocity fluctuations is expressed, accurate to the terms of order $(\tau_u/T_0)^2$, in terms of the carrier phase moments

$$\langle v_i(x,t)v_j(x,t)\rangle = f_{u1}\langle u_iu_j\rangle - \tfrac{1}{2}\tau_u(f_{u1}+f_{u2})\left(\frac{\partial\langle u_iu_j\rangle}{\partial t} + \langle V_k\rangle\frac{\partial\langle u_iu_j\rangle}{\partial x_k} + \langle u_iu_k\rangle\frac{\partial\langle V_j\rangle}{\partial x_k} + \langle u_ju_k\rangle\frac{\partial\langle V_i\rangle}{\partial x_k} + \frac{\partial\langle u_iu_ju_k\rangle}{\partial x_k}\right). \quad (19)$$

The first terms in expressions (17) and (19) describe the entrainment of particles into the pulsating motion of the carrier phase in a steady-state uniform turbulent flow [7, 10, 11] whereas the subsequent terms are linked with the nonuniformity and nonsteadiness of the turbulent flow. It follows from equations (17) and (19) for low-inertia particles ($\tau_u \to 0$) that $\langle u_iv_j\rangle \to \langle v_iv_j\rangle \to \langle u_iu_j\rangle$, i.e. the particle fluctuations coincide with the fluctuations of the carrier phase. For inertia particles $\langle u_iv_j\rangle \to 0$, but $\langle v_iv_j\rangle = O(\tau_u/T_0)$ because of the additional generation of solid phase fluctuations due to the nonuniformity of the carrying turbulent flow.

With the single-point moments and two-time functions of the carrier phase velocity and temperature fluctuations being known, equations (17)–(19) make it possible to calculate the second single-point moments that include discrete phase velocity and temperature fluctuations in a non-stationary non-uniform turbulent flow.

Equations (15) and (16) yield an expression for the two-time correlation of the solid and carrier phase velocity fluctuations accurate to the terms of the order of

$$\langle u_i(x,t)v_j(x,t+s)\rangle = \frac{\exp\left(\frac{s}{\tau_u}\right)}{\tau_u}\times\int_s^t \exp\left(-\frac{\xi}{\tau_u}\right)F_u(\xi)\,\mathrm{d}\xi. \quad (20)$$

The solid phase motion equation (9) and expression (12) include the correlation between the solid phase concentration and the liquid phase velocity fluctuations. To calculate this correlation, write the particle concentration balance equation (4) in integral form, multiply it by $u_i(x,t)$ and average over the ensemble of turbulent realizations; then, neglecting the terms proportional to the second and high-order derivatives of the averaged concentration, obtain the following expression:

$$\langle C(x,t)u_i(x,t)\rangle = -\int_0^t \langle u_i(x,t)v_k(x,s)\rangle\, \mathrm{d}s \frac{\partial \langle C(x,t)\rangle}{\partial x_k}.$$

With equation (20) taken into account, obtain

$$\langle C(x,t)u_i(x,t)\rangle = -\tau_u f_{u3}\langle u_i u_k\rangle \frac{\partial\langle C\rangle}{\partial x_k} \quad (21)$$

where

$$f_{u3} = \frac{1}{\tau_u}\int_0^\infty \left[1-\exp\left(-\frac{s}{\tau_u}\right)\right] F_u(s)\,\mathrm{d}s.$$

Using equation (21), write down the solid phase motion equation (9) in the following form:

$$\frac{\partial\langle V_i\rangle}{\partial t} + \langle V_k\rangle\frac{\partial\langle V_i\rangle}{\partial x_k} + \frac{\partial\langle v_i v_k\rangle}{\partial x_k} = \frac{1}{\tau_u}(\langle U_i\rangle - \langle V_i\rangle) - \frac{1}{\tau_u}D_{ik}\frac{\partial \ln\langle C\rangle}{\partial x_k} \quad (22)$$

where

$$D_{ik} = (f_{u1}+f_{u3})\tau_u\langle u_i u_k\rangle = \int_0^\infty F_u(s)\,\mathrm{d}s\langle u_i u_k\rangle$$

is the turbulent diffusion coefficient of admixture particles. Taking into account the interphase interaction term in equation (12), in which the correlation moments can be found from formulae (17) and (21), the equations for the second single-point moments of the carrier phase velocity fluctuations in the presence of particles will have the form

$$\left(1+f_{u2}\langle C\rangle\frac{\rho_2}{\rho_1}\right)\frac{\partial\langle u_i u_j\rangle}{\partial t} + \left(\langle U_k\rangle + \frac{\rho_2}{\rho_1}\langle C\rangle f_{u2}\langle V_k\rangle\right)\frac{\partial\langle u_i u_j\rangle}{\partial x_k}$$
$$+\langle u_i u_k\rangle\left(\frac{\partial\langle U_j\rangle}{\partial x_k} + \frac{\rho_2}{\rho_1}\langle C\rangle f_{u2}\frac{\partial\langle V_j\rangle}{\partial x_k}\right) + \langle u_j u_k\rangle\left(\frac{\partial\langle U_i\rangle}{\partial x_k} + \frac{\rho_2}{\rho_1}\langle C\rangle f_{u2}\frac{\partial\langle V_i\rangle}{\partial x_k}\right)$$
$$+\left(1+\frac{\rho_2}{\rho_1}\langle C\rangle f_{u2}\right)\frac{\partial\langle u_i u_j u_k\rangle}{\partial x_k}$$
$$= -\frac{1}{\rho_1}\left[\frac{\partial\langle u_i p\rangle}{\partial x_j} + \frac{\partial\langle u_j p\rangle}{\partial x_i}\right] - \left\langle\frac{p}{\rho_1}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\rangle + \nu_1\frac{\partial^2\langle u_i u_j\rangle}{\partial x_k \partial x_k} - 2\nu_1\left\langle\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle$$
$$-\frac{\rho_2}{\rho_1}\frac{\partial\langle C\rangle}{\partial x_k} f_{u3}\left[\langle u_i u_k\rangle(\langle U_j\rangle - \langle V_j\rangle) + \langle u_j u_k\rangle(\langle U_i\rangle - \langle V_i\rangle)\right]$$
$$-2\frac{f_{u4}}{T_E}\frac{\rho_2}{\rho_1}\langle C\rangle\langle u_i u_j\rangle \quad (23)$$

where

$$f_{u4} = \frac{T_E}{\tau_u}(1-f_{u1}). \quad (24)$$

As is seen from equation (23), the particles being entrained into the carrier phase pulsating motion contribute to the terms that determine the convective transfer and turbulent diffusion of liquid phase fluctuations and also the origination of fluctuations from the averaged motion; moreover, new terms appear that describe variations in pulsating energy due to the admixture concentration gradient [the last but one term in equation (23)] and additional dissipation of the carrier phase pulsating energy by particles (the last term).

In the case of inertia-free particles ($\tau_u \to 0$, $f_{u1}\to 1$, $f_{u2}\to 1$, $f_{u4}\to 0$) equations (23) for particles of constant concentration transform into equations for the second single-point moments of the fluctuations of a single-phase liquid having the density $\rho_1(1+\rho_2/\rho_1\langle C\rangle)$. Inertia particles ($\tau_u > T_E$) are less entrained into pulsating motion ($f_{u1}<1$, $f_{u2}<1$, $f_{u4}\neq 0$) and, as a result, the carrier phase turbulent energy may decrease. Large particles ($t_u \gg T_E$) are not entrained into pulsating motion and therefore f_{u1}, f_{u2}, $f_{u4}\approx 0$.

4. EQUATIONS FOR THE HYDRODYNAMICS OF TURBULENT GAS SUSPENSION FLOWS IN TUBES

To calculate the functions that describe the degree of the entrainment of particles into the carrier phase pulsating motion, it is necessary to determine the two-time correlation function of the carrier phase velocity fluctuations. To simplify the subsequent calculations, assume the function $F_u(s)$ to have the following form:

$$F_u(s) = \begin{cases}1, & \text{if } 0\leqslant s\leqslant T_p \\ 0, & \text{if } s > T_p\end{cases}. \quad (25)$$

Here T_p is the time of particle interaction with the turbulent field which is shorter than the life of energy-carrying vortices (turbulence time scale) because of the averaged and pulsating slip of the phases. By neglecting a decrease in T_p as compared with the lifetime of a turbulence mole, it is possible to assume in what follows that $T_p = T_E$. Thus, the main quantity which determines the degree of the entrainment of

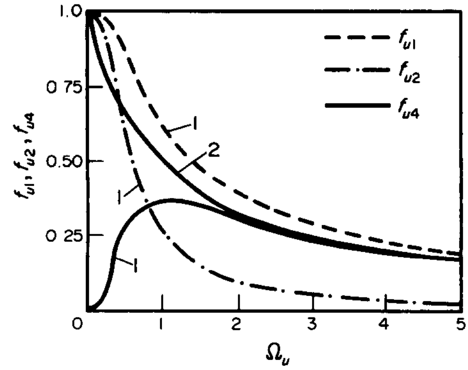

FIG. 1. Entrainment and dissipation coefficients vs the dynamic inertia parameter: 1, equations (18) and (24); 2, $(1+\Omega_u)^{-1}$ [23].

particles into turbulent motion is the particle dynamic inertia parameter $\Omega_u = \tau_u/T_E$.

Equations (18), (21) and (25) give expressions for the functions that describe the effect of particles on the turbulent structure of the carrying flow

$$f_{u1} = 1-\exp(-1/\Omega_u)$$
$$f_{u2} = 1-(1+1/\Omega_u)\exp(-1/\Omega_u)$$
$$f_{u3} = 1/\Omega_u+1-\exp(-1/\Omega_u)$$
$$f_{u4} = \exp(-1/\Omega_u)/\Omega_u. \quad (26)$$

Figure 1 presents the coefficients of entrainment f_{u1}, f_{u2} and dissipation f_{u4} vs the inertia parameter Ω_u plotted from formulae (26). It follows from these formulae that f_{u1}, $f_{u2} \to 1$ and $f_{u4} \to 0$ when $\Omega_u \to 0$, because the inertia-free particles are fully entrained into pulsating motion; f_{u1}, f_{u2}, $f_{u4} \to 0$ for $\Omega_u \to \infty$, because large particles are not entrained into pulsating motion. Figure 1 also contains the relation for the dissipation coefficient $f_{u4} = (1+\Omega_u)^{-1}$ [7, 23] obtained with the use of the correlation function $F_u = \exp(-s/T_E)$ which does not satisfy the condition $(\mathrm{d}F_u/\mathrm{d}s)_{s=0} = 0$—a fact which results in the condition $f_{u4} \to 0$ to be also invalid for $\Omega_u \to 0$ (of the inertia-free particles completely entrained into pulsating motion there should be no additional dissipation due to the pulsating slip of the energy-carrying moles of the carrier and solid phases). On the contrary, the relation for the dissipation coefficient $f_{u4} \sim \Omega_u$ for $\Omega_u < 1$ and $f_{u4} \sim$ const. for $\Omega_u > 1$ [11] tends to zero when $\Omega_u \to 0$, but it does not satisfy the limiting transition when $\Omega_u \to \infty$ (large particles are not entrained into pulsating motion and, consequently, there should not be additional dissipation by them either). The dissipation coefficient f_{u4} determined from formula (26) attains the maximum value at $\Omega_u = 1$, i.e. the turbulent energy dissipation due to the interphase pulsating slip reaches the maximum when the times of the particle relaxation and energy-carrying mole life coincide.

To describe the dissipation and transfer terms in equations (24) for the second single-point moments of velocity fluctuations of the particle-laden carrier phase, use is made of Rotta's approximation hypotheses [25]

$$\nu_1\left\langle\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle = \frac{c_E}{3}\frac{E^{3/2}}{L}\delta_{ij}+\frac{c_{1E}}{3}\frac{\langle u_i u_j\rangle}{L^2}$$
$$-\left\langle\frac{P}{\rho_1}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right\rangle = k_1\frac{E^{1/2}}{L}(\langle u_i u_j\rangle-\tfrac{2}{3}E\delta_{ij}). \quad (27)$$

The turbulence time scale (the lifetime of energy-carrying fluctuations) incorporated into the definition of the dynamic inertia parameter of particles is calculated from the formula $T_E = \gamma L/E^{1/2}$. The constant γ is estimated from the condition of equality of the mole turbulent energy E to the turbulent dissipation of pulsating energy for the time T_E, i.e. $E = c_E E^{3/2}/LT_E$, which results in $\gamma = 1/c_E$.

In ref. [26], a qualitative investigation is made into the effect of particles on the intensity of turbulent transfer of the gas suspension flow momentum. For large turbulent Reynolds numbers $Re_E = LE^{1/2}/\nu_1 \gg 1$ an algebraic set of equations is obtained in a diffusion-free approximation for the second single-point moments of particle-laden carrier gas velocity fluctuations, which yielded the expressions for the turbulent energy and carrier phase shear stress in the form of modified Prandtl numbers. The analysis of these expressions shows that inertia particles ($\Omega_u \sim 1$) cause a decrease in both the intensity of the turbulent fluctuations of gas due to the forces of resistance in the pulsating slip of phases. As the dynamic inertia parameter of particles increases ($\Omega_u \gg 1$), the effect of the admixture on the pulsating structure diminishes. With an increase in the weight concentration the low-inertia particles intensify the momentum turbulent transfer due to the participation of particles in the pulsating motion of the carrier phase and to the growth of the terms that describe the origination of gas turbulent fluctuations from averaged motion in the presence of particles.

Numerical calculation of the hydrodynamics of gas suspension flows in circular tubes is carried out with coinciding averaged velocities of the solid and carrier phases for constant concentration of the admixture over the tube cross-section. Use is made of the momentum balance equation for gas suspension flow in the axial direction

$$\frac{1}{\rho_2}\frac{\partial\langle P\rangle}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left\{r[\nu_1+\nu_{t1}+\langle\phi\rangle\nu_{t2}]\frac{\partial\langle U\rangle}{\partial r}\right\} \quad (28)$$

and the balance equation for the carrier phase pulsating energy in which the following gradient representation is employed for the terms describing turbulent diffusion:

$$\nu_{t1}\left(\frac{\partial\langle U\rangle}{\partial r}\right)^2(1+\langle\phi\rangle f_{u2})$$
$$+\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\nu_1+\frac{\alpha}{2}E^{1/2}L(1+\langle\phi\rangle f_{u2})\right]\frac{\partial E}{\partial r}\right\}$$
$$-\left(c_E+\langle\phi\rangle\frac{f_{u4}}{\gamma}\right)\frac{E^{3/2}}{L}-c_{1E}\nu_1\frac{E}{L^2}=0. \quad (29)$$

The turbulent viscosity of the solid phase is determined from equation (19) in terms of the turbulent kinematic viscosity of the carrier phase as

$$\nu_{t2}=-\frac{\langle v_x v_y\rangle}{\partial\langle U\rangle/\partial y}=\nu_{t1}f_{u1}\left[1+\frac{\Omega_u(f_{u2}+f_{u1})}{2f_{u1}}\times k_1\gamma\frac{1+2\langle\phi\rangle f_{u4}/(k_1\gamma)}{1+f_{u2}\langle\phi\rangle}\right]. \quad (30)$$

The turbulent viscosity of the liquid phase is determined from the set of equations (23) in diffusion-free approximation with the use of Rotta's relations (27)

$$\frac{\nu_{t1}}{\nu_1}=\alpha_1 Re_E\frac{1+f_{u2}\langle\phi\rangle}{\left(1+\dfrac{\beta}{Re_E}+\dfrac{2f_{u4}\langle\phi\rangle}{k_1\gamma}\right)}. \quad (31)$$

The turbulent macroscale L is identified with the Nikuradze mixing length

$$L=0.14-0.08r^2-0.06r^4.$$

Numerical values of the constants α_1, β, k_1, c_E and c_{E1} are selected from the solution to the problem of a near-wall single-phase liquid flow [27]: $\alpha_1=0.51$, $\beta=14$, $k_1=1.16$, $c_E=0.13$, $c_{E1}=0.32$. The boundary conditions for the set of equations (28)–(31) take the form

$$E=\langle U\rangle=0,\quad \text{if}\quad r=R;$$
$$\frac{\partial\langle U\rangle}{\partial r}=\frac{\partial E}{\partial r}=0,\quad \text{if}\quad r=0. \quad (32)$$

The set of equations (28)–(31) with boundary conditions (32) are solved numerically.

5. CALCULATION RESULTS

The admixture of solid particles considerably changes the intensity of the carrier phase pulsating motion. Figure 2 illustrates the distribution of the gas turbulent energy intensity. It is seen that the admixture of solid particles can both increase the intensity of gas turbulent fluctuations, because of the growth of fluctuations from the averaged motion when the particles become entrained into turbulent motion, and decrease the level of turbulent fluctuations of gas through the work done by the carrier phase to entrain the admixture mass into pulsating motion. The degree of the effect of particles on the gas fluctuation structure depends nonmonotonously on the particle dynamic inertia parameter as is seen from Fig. 3 where the distribution of the gas pulsating energy in the near-wall region is shown. A maximum reduction in the gas pulsating energy is achieved when the dynamic inertia parameter is about unity. The particle dynamic inertia parameter changes across the channel; it attains its maximum near the channel wall where the frequency of turbulent fluctuations is a maximum. Closer to the flow core the dynamic inertia parameter decreases. It should be noted that when the admixture mass concentration increases, the carrier phase turbulent flow adjusts itself so as to diminish the effect of admixture on the fluctuation structure of the flow; the dynamic inertia parameter of low-inertia particles ($\Omega_u \ll 1$) increases with the admixture concentration, decreasing the degree of the additional growth of fluctuations from averaged motion and increasing the pulsating slip of phases, while the inertia parameter of larger particles ($\Omega_u > 1$) decreases in a flow with a higher concentration of the admixture resulting in a smaller additional dissipation of the gas turbulent

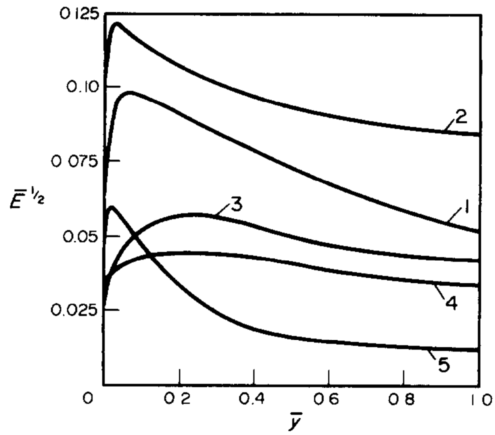

FIG. 2. Effect of particles on the pulsating energy of the carrying gas ($Re = 5.3\times10^4$): 1, $\langle\phi\rangle = 0$; 2, $\langle\phi\rangle = 5$, $R/a = 30\,000$; 3, $\langle\phi\rangle = 5$, $R/a = 5000$; 4, $\langle\phi\rangle = 5$, $R/a = 3000$; 5, $\langle\phi\rangle = 5$, $R/a = 1000$.

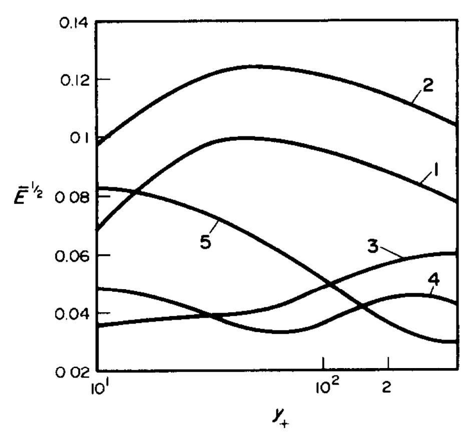

FIG. 3. Effect of discrete phase on the fluctuation structure of gas near the wall ($Re = 5.3\times10^4$). For notation, see Fig. 2.

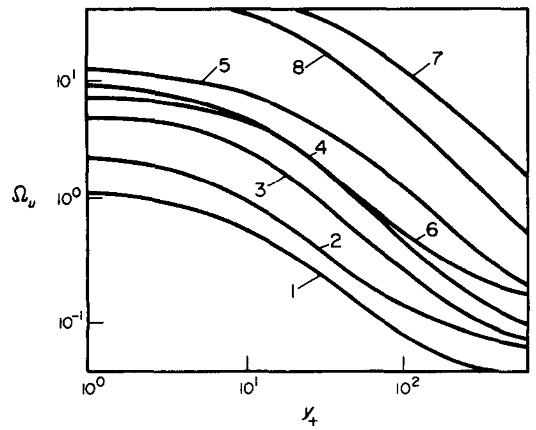

FIG. 4. Variation of the admixture particle inertia parameter over the tube cross-section ($Re = 5.3 \times 10^4$): 1, $\langle\phi\rangle = 0$, $R/a = 10\,000$; 2, $\langle\phi\rangle = 5$, $R/a = 10\,000$; 3, $\langle\phi\rangle = 0$, $R/a = 5000$; 4, $\langle\phi\rangle = 5$, $R/a = 5000$; 5, $\langle\phi\rangle = 0$, $R/a = 3000$; 6, $\langle\phi\rangle = 5$, $R/a = 3000$; 7, $\langle\phi\rangle = 0$, $R/a = 1000$; 8, $\langle\phi\rangle = 5$, $R/a = 1000$.

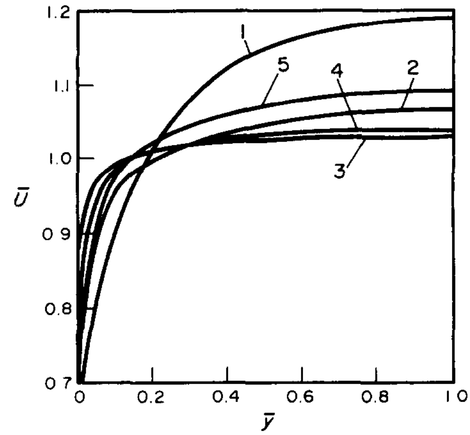

FIG. 6. Effect of discrete phase on the gas averaged velocity profile ($Re = 5.3 \times 10^4$): 1, $\langle\phi\rangle = 0$; 2, $\langle\phi\rangle = 3$, $R/a = 30\,000$; 3, $R/a = 5000$; 4, $\langle\phi\rangle = 3$, $R/a = 30\,000$; 5, $\langle\phi\rangle = 3$, $R/a = 1000$.

energy and in a more intensive generation of the carrier phase fluctuations (Fig. 4).

The turbulent viscosity of gas, just as its pulsating energy, depend on the mass concentration of the admixture. Low-inertia particles ($\Omega_u \ll 1$) increase the turbulent viscosity of gas. On the other hand, in the presence of inertia particles ($\Omega_u > 1$) a reduction in the carrier phase turbulent viscosity is observed ($\langle\phi\rangle < 5$), followed by an increase of turbulent viscosity with the admixture concentration (Fig. 5). The non-monotonous behaviour of ν_{tl} caused by the admixture mass concentration is supported by experimental data presented in ref. [3].

The particles exert an influence not only on the fluctuation structure of a flow, but also change the profile of the averaged velocity of the carrier phase (Fig. 6). Addition of fine particles flattens the profile of the gas averaged velocity. As the particle dynamic inertia parameter Ω_u increases, the effect of particles on the averaged velocity profile diminishes and this is supported by experimental investigations of the gas averaged velocity profile in a gas suspension flow [28].

Figure 7 gives a comparison between experimental [19] and calculated relations for the hydraulic resistance of an air flow laden with zinc particles of different sizes. There is a good agreement between the experimental data and the predicted results, especially for fine particles. In the case of larger particles, there is no reduction in the hydraulic resistance with an increase in the admixture mass concentration. This discrepancy between the prediction and experiment

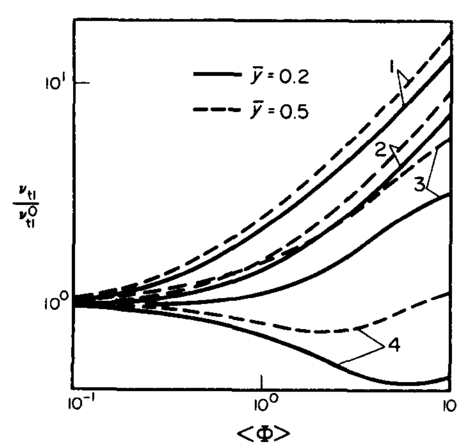

FIG. 5. Effect of particles on the kinematic viscosity of gas ($Re = 5.3 \times 10^4$): 1, $R/a = 30\,000$; 2, $R/a = 5000$; 3, $R/a = 300$; 4, $R/a = 1600$.

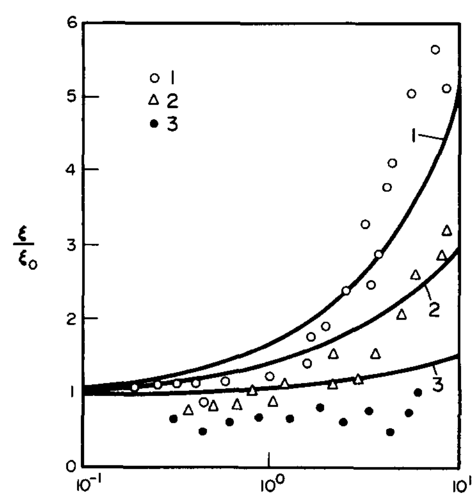

FIG. 7. Hydraulic resistance of gas suspension flow in a circular tube ($Re = 5.3 \times 10^4$): 1, $R/a = 30\,000$; 2, $R/a = 5000$; 3, $R/a = 3400$; 4, $R/a = 1700$.

can be attributed to several reasons. First, the calculations were carried out for the particle concentration being constant over the channel cross-section. Second, the proposed model does not take into account the effect of the discrete admixture on the turbulent scale. Third, the expressions for the second single-point moments of the discrete phase velocity fluctuations have been derived in the 'local equilibrium' approximation when it is assumed that the intensity of the pulsating motion of particles at the selected point in the flow is determined by the intensity of the liquid phase turbulent fluctuations at this very point. This assumption is valid for low-inertia particles; larger particles require their finite inertia path taken into account, the magnitude of which can be compared with the characteristic scale of variation in the pulsating and averaged characteristics of a turbulent flow. It should be noted that calculations made without the turbulent flow inhomogeneity being included in the expressions for the second single-point moments of the solid and carrier phase velocity fluctuations, equations (17) and (19), almost do not display an increase in the hydraulic resistance on an increase of the admixture mass concentration even for small particles; for larger particles the calculations give a monotonous reduction in the hydraulic resistance of the gas suspension flow on an increase of admixture mass—the fact which does not agree with the experimental data.

6. CONCLUSIONS

(1) Using the method of averaging over the ensemble of turbulent flow realizations, the mass and momentum balance equations are obtained for the solid phase and the flow as a whole. The equations show that the entrainment of particles into pulsating motion increases turbulent Reynolds stresses of a gas suspension flow.

(2) Closed expressions are obtained for the second single-point moments of fluctuations of the solid and carrier phase characteristics in terms of the carrier phase velocity fluctuations with the nonuniformity and unsteadiness of the carrying turbulent flow taken into account.

(3) Without resorting to additional constants associated with the presence of particles in the flow, hydrodynamic calculations for the gas suspension flow in circular tubes are made. It has been found that depending on the particle dynamic inertia parameter, the carrier phase turbulent fluctuations may increase ($\Omega_u \ll 1$) or decrease ($\Omega_u > 1$). The calculations showed the flattening of the gas averaged velocity profile in the presence of particles; the non-monotonous relationship between the carrier phase turbulent viscosity and admixture mass concentration is investigated. Comparison of the predicted results for the gas suspension flow hydraulic resistance with experimental data for circular tubes indicates a satisfactory description of momentum transfer by a dust-laden flow.

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HYDRODYNAMIQUE ET TRANSFERT THERMIQUE DES ECOULEMENTS TURBULENTS GAZEUX AVEC SUSPENSION DANS DES TUBES—1. HYDRODYNAMIQUE

Résumé—Par la méthode de moyenne sur l'ensemble des réalisations d'écoulements turbulents, on écrit les équations du mouvement moyen et du bilan de masse pour la phase solide et pour le fluide considérés comme un tout. Des expressions de fermeture pour les moments de second ordre au même point des fluctuations du solide et du porteur sont obtenues en fonction des corrélations des fluctuations de la phase vectrice dans un écoulement turbulent non homogène. A partir de ces expressions, un système d'équations est écrit pour les moments de second ordre au même point des fluctuations de vitesse de la phase vectrice en présence des particules. Une analyse hydrodynamique est présentée pour les écoulements turbulents gazeux avec suspension dans des tubes. La comparaison avec des données expérimentales montre une description satisfaisante des mécanismes de transfert de quantité de mouvement par un écoulement chargé de particules.

HYDRODYNAMIK UND WÄRMETRANSPORT BEI TURBULENTER STRÖMUNG EINER GASSUSPENSION IM ROHR—1. HYDRODYNAMIK

Zusammenfassung—Durch das Verfahren der Ensemble-Mittelung in einer turbulenten Strömung werden die Mittelwert-Gleichungen für Impuls- und Massenerhaltung der festen Phase und der Strömung insgesamt abgeleitet. Es werden geschlossene Ausdrücke für das zentrale Moment der Flukuationen für Feststoff und Trägergas ermittelt, und zwar in Form von Korrelationen der Geschwindigkeits-Fluktuationen im Trägergas bei nicht-homogener turbulenter Strömung. Auf diesen Ausdrücken aufbauend, wird ein Gleichungssystem für das zweite Moment der Fluktuationen der Trägerphasengeschwindigkeit in Gegenwart von Feststoffteilchen erstellt. Die turbulente Strömung einer Gassuspension im Rohr wird hydrodynamisch untersucht. Vergleiche mit experimentellen Daten zeigen eine zufriedenstellende Beschreibung des Impulstransports in einer partikelbeladenen Strömung.

ГИДРОДИНАМИКА И ТЕПЛООБМЕН ПРИ ТУРБУЛЕНТНОМ ТЕЧЕНИИ ГАЗОВЗВЕСИ В ТРУБАХ—1. ГИДРОДИНАМИКА

Аннотация—Методом осреднения по ансамблю реализаций турбулентного потока получены уравнения осредненного движения и баланса массы твердой фазы и потока в целом. Найдены замкнутые выражения для вторых одноточечных моментов пульсаций характеристик твердой и несущей фаз через корреляции пульсаций скорости несущей фазы в неоднородном турбулентном потоке. На основе полученных выражений записана система уравнений для вторых одноточечных моментов пульсаций скорости несущей фазы в присутствии частиц. Проведены расчеты гидродинамики при турбулентном течении газовзвеси в трубах. Сравнение с экспериментальными данными свидетельствует об удовлетворительном описании процессов переноса импульса запыленным потоком.